Examples 3 in Basic Geometry

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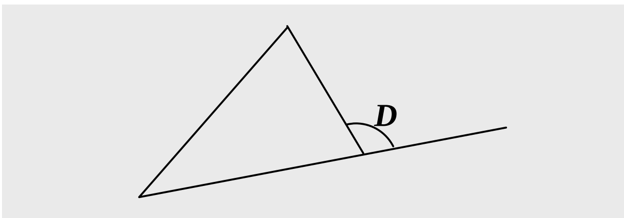
Examples 3

0. The sum of all the angles in a triangle is 180^o. What then about the sum of all the angles in each of the polygons as follows. tetragon, pentagon, hexagon, and a polygon with 151 sides?

1. In a polygon, an external (exterior) angle is the angle that is supplement to an internal angle in the polygon, and is adjacent to the internal angle. And if two angles X and Y are supplement to each other, we get $X + Y = \mathbf{180^o}$. So for instance, the angle D below is an external angle in the triangle below.



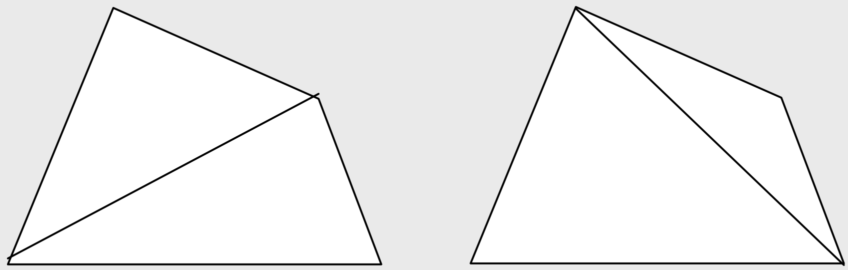
Find the sum of all the external angles for each of the polygons as follows. triangle, tetragon, pentagon, and a polygon with n sides.

Suggestions or Solutions
To the Problem 0

The sum of all the angles in a triangle is 180^{o}. What then about the sum of all the angles in each of the polygons as follows. tetragon, pentagon, hexagon, and a polygon with 151 sides?

Beginning with a tetragon, we can partition it into triangles in two ways below.

Fig. 0.0

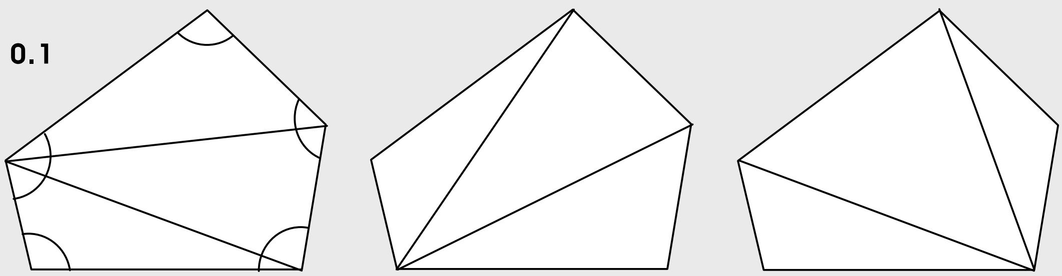


That is, putting together two triangles, we can make a tetragon. Each triangle has three angles, the sum of which is 180^{o}. What then is the sum of all the angles in a tetragon?

It is the sum of all the angles in two triangles. So the sum is $\mathbf{2 \cdot 180^{o}} = \mathbf{360^{o}}$.

Next, moving on to a pentagon, we can partition it into triangles many ways.

Fig. 0.1

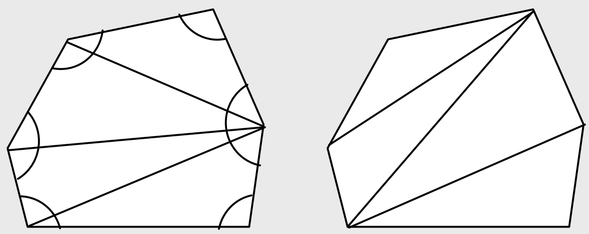


So putting together three triangles, we can make a pentagon. Each triangle has three angles, the sum of which is 180^{o}. What then is the sum of all the angles in a pentagon?

It is the sum of all the angles in three triangles. So the sum of all the angles in a pentagon is $\mathbf{3 \cdot 180^o} = \mathbf{540^o}$.

Next, moving on to a hexagon, we can also partition it into triangles. So putting together triangles, we can make a hexagon. How many triangles then have to be put together?

Fig. 0.2



So making a hexagon, we need put together at least four triangles, in each of which, the sum of the thee angles is 180^o.

Thus, the sum of all the angles in a hexagon is $\mathbf{4 \cdot 180^o} = \mathbf{720^o}$.

What then about the polygon with 151 sides?

We know these:

A tetragon has four sides, and can be made of two triangles.

A pentagon has five sides, and can be made of three triangles.

A hexagon has six sides, and can be made of four triangles.

How many triangles than do we need to put together to make a heptagon, which is a polygon with 7 sides?

Making a heptagon, we need put together at least five triangles. So the sum of all the angles in a heptagon is $\mathbf{5 \cdot 180^o}$, which is $\mathbf{900^o}$.

And we can say that making an octagon, which is a polygon with 8 sides, we need put together at least six triangles. So the sum of all the angles in an octagon is $\mathbf{6 \cdot 180^o}$, which is $\mathbf{1080^o}$.

What then about the polygon with $\boldsymbol{n}$ sides?

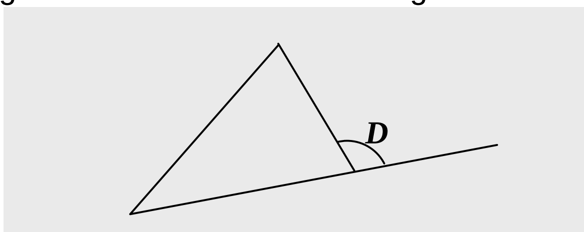
Making a polygon with $\boldsymbol{n}$ sides, we need put together at least $(\boldsymbol{n} - \mathbf{2})$ triangles.

So the sum of all the angles in a polygon with $\boldsymbol{n}$ sides is $\mathbf{(}\boldsymbol{n}\mathbf{ - 2)\cdot 180^o}$.

Thus, the sum of all the angles in a polygon with 151 sides is $\mathbf{149 \cdot 180^o} = \mathbf{26820^o}$.

Suggestions or Solutions
To the Problem 1

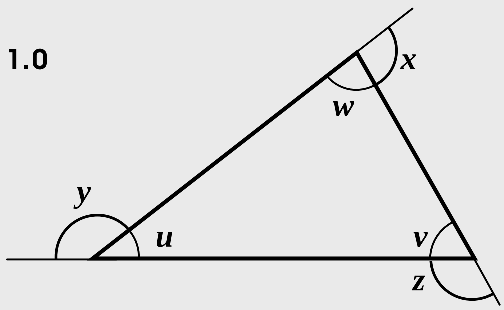
In a polygon, an external (exterior) angle is the angle that is supplement to an internal angle in the polygon, and is adjacent to the internal angle. And if two angles ***X* and *Y*** are supplement to each other, we get $X + Y = 180^o$. So for instance, the angle ***D*** below is an external angle in the triangle below.



Find the sum of all the external angles for each of the polygons as follows. triangle, tetragon, pentagon, and a polygon with ***n*** sides.

We know if two line segments are in a line, the angle between the two is 180^o.

Fig. 1.0

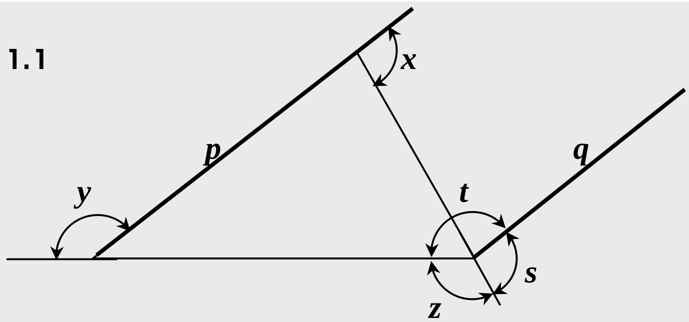


And we know $x + y + z = 180^o$.

So the sum is $3 \cdot 180^o - 180^o = 360^o$.

And we know that the corresponding angles are the same, so are the alternate angles. So if in Fig. 1.1, ***p*** and ***q*** are parallel to each other, we get $x = s$, and $y = t$.

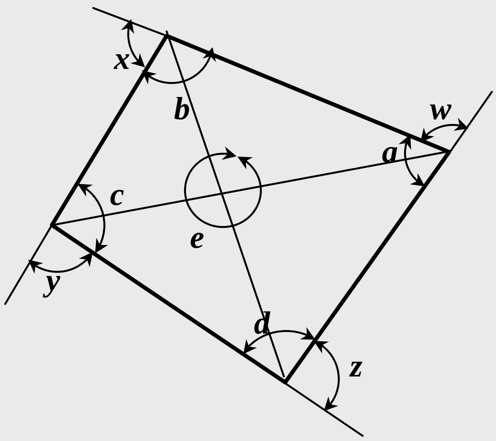
Fig. 1.1



That's because ***x*** and ***s*** are corresponding angles, and so are ***y*** and ***t***.
Thus, the sum is $x + y + z = s + t + z = 360^o$.

Next, moving on to a tetragon, we can partition it into four triangles the way below.

Fig. 1.2



Then first, we can say that $e = 360^o$, and $a + b + c + d + e = 4 \cdot 180^o$, because the three angles in a triangle add up to 180^o, and the tetragon is made of four triangles.

So we get $a + b + c + d = 2 \cdot 180^o$.

And next, we can say that $(a + w) + (b + x) + (c + y) + (d + z) = 4 \cdot 180^o$.

In other words, we have $(a + b + c + d) + (w + x + y + z) = 4 \cdot 180^o$.

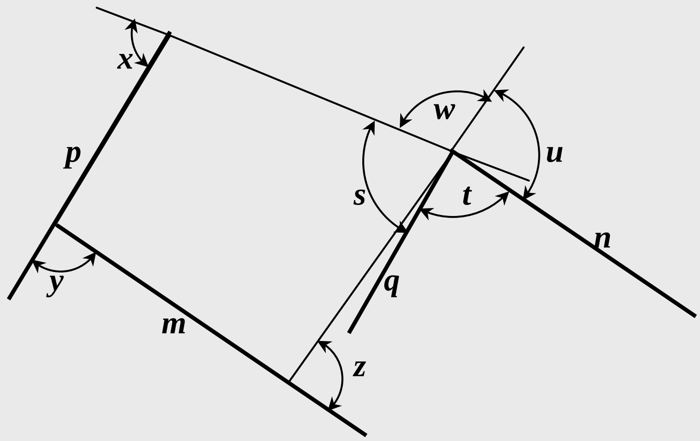
And we know $a + b + c + d = 2 \cdot 180^o$.

Thus, we get $w + x + y + z = 2 \cdot 180^o = 360^o$.

So in the case of a tetragon, too, the sum of all the external angles is 360^o.

And we know that the corresponding angles are the same, so are the alternate angles. So assuming in the figure below, p and q are parallel to each other, and so are m and n, we get $x = s$, $y = t$, and $z = u$.

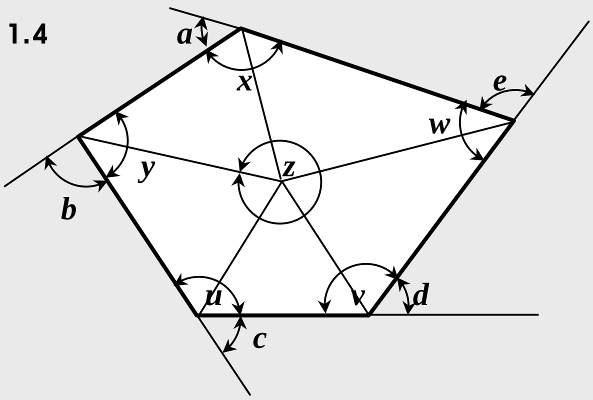
Fig. 1.3



That's because x and s are corresponding angles, and so are y and t, and the same is true for z and u, too. Thus, the sum is $w + x + y + z = w + s + t + u = 360^o$.

Next, moving on to a pentagon, we can partition it the way below.

Fig. 1.4



Then first, we can say that $z = 360^o$, and $x + y + z + u + v + w = 5{\cdot}180^o$, because the three angles in a triangle add up to 180^o, and the pentagon is made of five triangles.

So we get $x + y + u + v + w = 3{\cdot}180^o$.

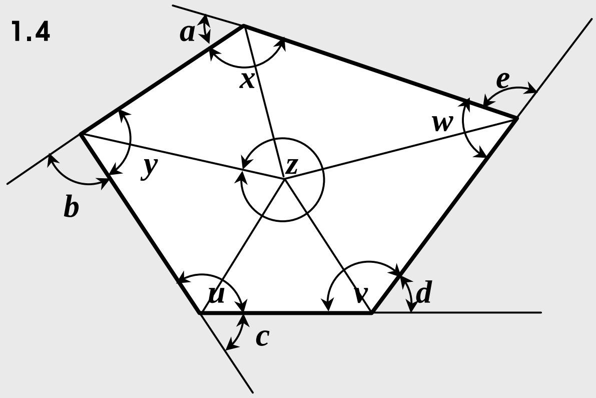
And next, we can say that $(a + x) + (b + y) + (c + u) + (d + v) + (e + w) = 5{\cdot}180^o$.

In other words, we have $(a + b + c + d + e) + (x + y + u + v + w) = 5{\cdot}180^o$.

And we know $x + y + u + v + w = 3{\cdot}180^o$. Thus, we get $a + b + c + d + e = 2{\cdot}180^o$.

So in the case of a pentagon, too, the sum of all the external angles is 360^{o}.

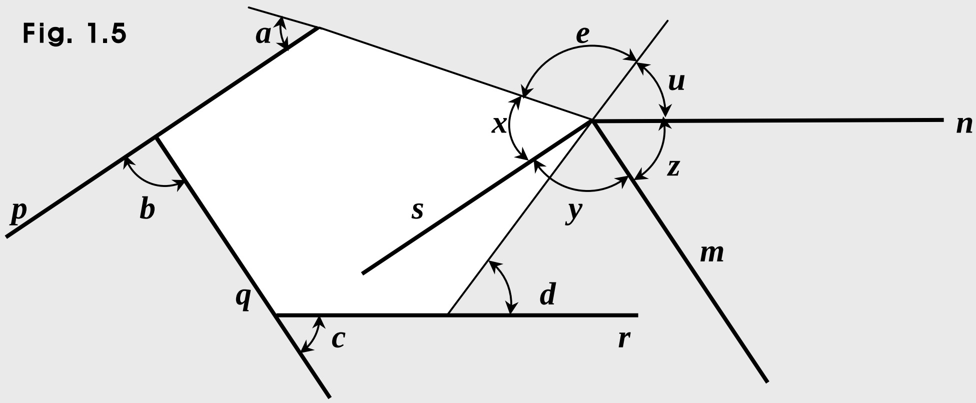
Fig. 1.4



And we know that the corresponding angles are the same, so are the alternate angles.
So as in Fig. 1.5 below, let's now add some line segments to Fig. 1.4 above so that ***p*** and ***s*** are parallel, ***q*** and ***m*** are parallel, and ***r*** and ***n*** are parallel.

Then, we get $a = x$, $b = y$, $c = z$, and $d = u$.

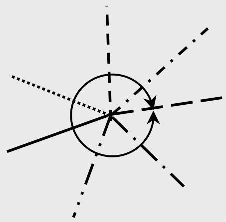
Fig. 1.5



It's because ***a*** and ***x*** are corresponding angles, and the same is true, also, for ***b*** and ***y***, for ***c*** and ***z***, and for ***d*** and ***u***.
Thus, the sum is $a + b + c + d + e = x + y + z + u + e = 360^{o}$.

Suppose next, seven line segments meet each other at one point the way below.

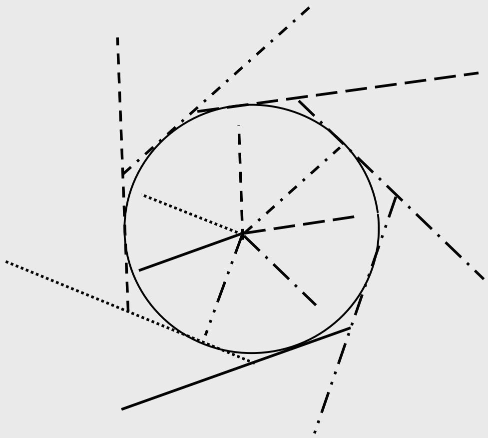
Fig. 1.6



Then, the sum of all the angles between the line segments is 360°.

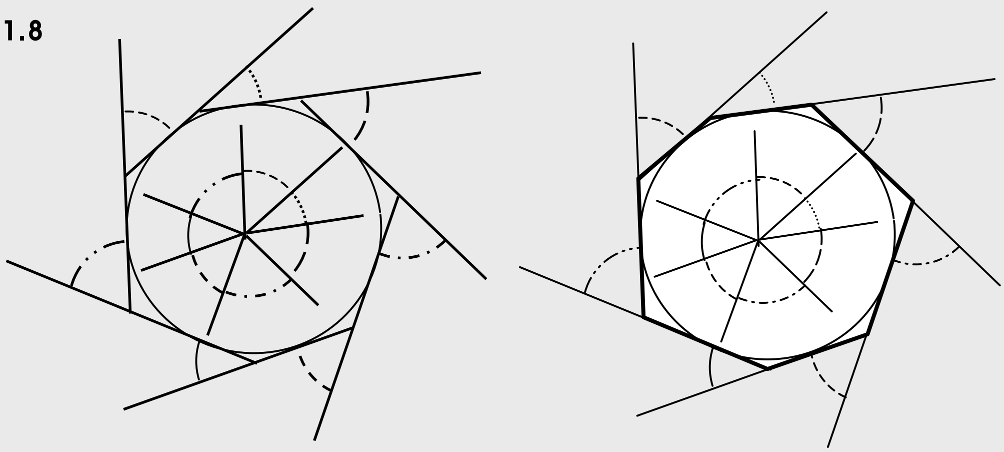
Let’s next, put line segments along a circle the way below so that each line segment is tangent to the circle, and is parallel to one of the seven line segments above.

Fig. 1.7



Then, we can indicate pairs of corresponding angles the way below.

Fig. 1.8



Then, we can see a heptagon, and can say that the sum of all the external angles of a heptagon is 360^o.

Thus, we can conclude that taking the sum of all the external angles in every polygon, we get 360^o. That is to say that in a polygon with $\boldsymbol{n}$ sides, all the external angles add up to 360^o.